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Referee's report on the doctoral dissertation of Syed Naqvi
"Exact standing wave spacetimes"

Introduction

Gravitational waves are currently one of the most quickly developing areas of research in general relativity. This is largely due to the progress in the field of gravitational wave detectors and the first detections of gravitational wave outbursts from merging binaries we have observed in the recent decade. Even though the problem of gravitational wave propagation dates back to the earliest years of general relativity, many important theoretical problems connected to the emission and propagation of gravitational waves are still open. I was therefore very happy to be able to report on a thesis related to this fascinating topic when I was offered the opportunity to do so.

The dissertation of Syed Naqvi is concerned with the problem of exact gravitational wave solutions of the Einstein equations corresponding to standing waves. In the simplest linear approximation it is relatively easy to define and construct solutions with standing waves by superimposing waves propagating in opposite directions. In the non-linear regime, however, both the definition and the construction of examples is much more difficult. Syed Naqvi investigated 3 examples of exact solutions, belonging to well-known classes of Einstein-Rosen and Gowdy metrics.

Syed Naqvi focused on the geometry and the properties of time-like geodesics in these spacetimes. He investigated three types of solutions corresponding to gravitational waves in vacuum and in the presence of electromagnetic waves.

Description of the dissertation

The dissertation is based on two articles published in Physical Review D and some additional unpublished results. It consists of 5 chapters. In the first chapter the author introduces the topic, presents previous results and discusses the notation, the main themes and the mathematical methods used in this work. These include the physics of standing waves, basics of general relativity, Newman-Penrose formalism and theory of dynamical systems and deterministic chaos. The chapter is fairly long, but also relatively well-written and provides a good starting point for the rest of the work.

In the second chapter the author considers vacuum gravitational wave solutions with standing waves with 3-torus topology. The metric can be analyzed either as a T^3 Gowdy spacetime or as an axially symmetric Einstein-Rosen wave. Interestingly, the author also considers non-linear effects of gravitational waves, including the longitudinal effects of a gravitational wave. Both the geodesic equation and the geodesics deviation equations have been investigated, as well as the curvature in the Newman-Penrose decomposition.

The third chapter deals with an electro-vacuum solution, i.e. a standing wave containing both a gravitational and an electromagnetic standing wave. The results seem to match the vacuum case fairly well.

In the fourth chapter Syed Naqvi discusses a cylindrical standing wave. The main focus is on the behavior of test masses and the chaotic behavior of geodesics investigated by the means of numerical integration of the equations of motion. The author discusses first the double pendulum as a toy model for comparison. Then the author applies the machinery of chaos theory, including the Poincare maps and theory of fixed points and their invariant submanifolds, to prove that many geodesics exhibit chaotic behavior. In particular, Naqvi identifies the emergence of a chaotic tangle, i.e. a situation in which the stable and unstable invariant submanifolds of hyperbolic fixed points cross leading to unstable, chaotic behavior of the geodesics. The author manages to decipher the geometric structure of the tangle.

The fifth chapter, much shorter than the previous ones, contains the Author’s summary and outlook. This is followed by the list of references and an appendix with additional definitions from basic topology.

Assessment of the quality of the dissertation

I find Syed Naqvi’s dissertation very interesting. His results provide new examples of chaotic systems in the field of general relativity. The behavior of geodesics gives a simple physical description of the standing wave spacetimes in a covariant way. While the toy models he investigated are rather unlikely to describe any astrophysical sources, they are of considerable theoretical interest. Numerical methods of investigation Syed Naqvi has employed seem to be the only viable ones in these spacetimes.

I would also like to commend the author’s style. The dissertation is written in a very readable way and I found it very easy to digest. Naqvi avoids the usual style of dissertation writing in mathematical physics, with the results presented in the form theorems, more typical for mathematical writing, opting for a more colloquial and direct approach. I believe this suits this type of work much better.

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I have found a small number of problems, mainly typos:

1. On page 51: letter d missing in eq. 2.1
2. On page 54: “odd and even values of k lead to different values of the metric functions, but without loss of generality we may assume k=0” - can you explain this in detail?
3. On page 75: the number of the equation referred to in the second paragraph is missing
4. On page 99: “dark blue” referring to Fig. 4.3 should probably read “purple”

Apart from that I have a couple of questions regarding the results.

1. The author never considers null geodesics, or light rays. While this is strictly speaking outside the topic of the dissertation, I wonder if the author believes that they would behave qualitatively differently from the timelike ones?
2. Is there something like a threshold amplitude of the gravitational waves beyond which we may expect chaos?
3. Is it possible to use the geodesic deviation equation in cylindrical standing wave spacetime? Would this lead to any new insights regarding spacetime?
4. Wouldn’t a periodically driven rotator (for example, a kicked rotator) be a better analogy to a particle in a standing wave then a double pendulum? Standing waves are strictly periodic in time, providing this way periodic “kicks” to particles, in full analogy to the periodically driven rotator.

Final conclusion

I conclude that the presented dissertation meets the formal requirements for a PhD thesis, and I recommend the admission of the Candidate to the subsequent stages of the procedure, including the public defense.

Mikołaj Korzyński

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