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Report on the Ph.D. thesis
Exact Standing Wave Spacetimes
by **Mr. Syed Umair Husain Naqvi**

The extensive Ph.D. thesis by Mr. Syed Umair Husain Naqvi, titled **Exact Standing Wave Spacetimes**, not only presents the calculations and results from his two papers co-authored with his supervisor, Prof. Sebastian Szybka, but also offers significant extensions to their findings. Both papers have been published in Physical Review D, a leading journal in the field of the classical and quantum theory of gravitation. The thesis comprises an introduction outlining the discussed problem set, three main chapters presenting original results, and concluding remarks. In addition to requisite declarations of originality and acknowledgments, the thesis includes one appendix and an extensive reference list containing 201 entries. The structure of the thesis reflects three (mutually related) research projects undertaken by the author, two of which have already been published. They are (in the order adopted in the thesis):

1. Analysis of the standing waves in the T^3 Gowdy spacetimes with a special emphasis on the motion of test particles.

2. An attempt to extend a similar analysis to the case of the standing electromagnetic wave coupled with the standing gravitational wave.

3. Analysis of the chaotic motion of test particles in the spacetime of the cylindrical standing gravitational wave.

Below, I will briefly describe the content of each chapter, with a special emphasis on the original results. In the first - introductory - chapter of the thesis, the author gives a short

exposition of general relativity, its basic equations and methods, introduces the reader to the concept of the standing waves in general and gives some basic information on the dynamical systems and chaos. It also contains short information on the differentiable manifolds, (the definition of the topological space is relegated to the appendix), maps, structures, covariant derivatives and important (from the point of view of physical applications) tensors. It also introduces the Newman-Penrose formalism and Petrov classification. Although time-consuming to prepare, including such an introductory chapter in the thesis should be considered a very good practice. It undoubtedly assists prospective readers in placing the presented results within a broader context. Furthermore, such an introduction defines the prerequisites necessary for a proper understanding of the techniques and results presented in the thesis.

The objective of Chapter 2 (which is based on the article *Freely falling bodies in a standing-wave spacetime*, Phys. Rev. D **103**, (2021) 024011) is to analyze in some detail the motion of the test particles in the spacetime of standing waves described by the three-torus Gowdy model. The idea is to carefully investigate the geodesics with a special emphasis on their behaviour in the vicinity of the antinodes. All the results are presented graphically. The author starts with the simplest two-wave case and solves numerically equations describing 31 geodesics. In Fig. 2.11 the formation of the antinode is visible. As the number of waves increases so does the number of antinodes and to understand better the nature of the formation of the antinodes (or crossing of the geodesics at the antinodes) Mr. Naqvi employs the Newman-Penrose formalism. As expected, the tidal forces seem to attract test particles towards the antinodes. The attempts to relate the behaviour of the test particles at the antinode with the concentration of energy is of course inconclusive because of the lack of a unique concept of the gravitational energy.

In Chapter 3 (the shortest in the Thesis) Mr. Naqvi extends the analyses of the vacuum T^3 Gowdy solution to the eletrovacuum T^3 Gowdy solution. The results are presented only briefly as if the author were unsure of their significance. Therefore, I will limit myself to only enumerating what has been done. First, after choosing the line element, Mr. Naqvi constructs the geodesic equations and numerically investigates 31 geodesics. I must admit that I do not know what the principal results are and, in my opinion, the way the material is presented is somewhat pessimistic. The next subsection entitled 'Geodesic deviation equation' contains a few equations and practically no discussion. It seems to me that this project is still in its early stages.

In Chapter 4 (which is based on the article *Chaos and Einstein-Rosen gravitational waves*, Phys. Rev. D **108** (2023), L081501) Mr. Naqvi attempts to answer an important question if the chaotic behavior of geodesics is typical in the Einstein-Rosen spacetime and if

there appears a fractal structure? To answer this question the author analyses the sensitivity of the geodesics to a choice of the initial conditions by studying chaotic orbits. Further, he investigates the fractal structure in a similar manner. To gain a better understanding of the problem Mr. Naqvi studies the Poincare sections. This chapter is based on heavy numerical calculations and the results are presented graphically. Unfortunately, only a few very general observations are given.

In Chapter 5, Mr. Naqvi gives a short description of each chapter of the thesis and poses some general questions, which, may indicate directions for further studies.

The thesis is well-written and has a fairly clear structure: Chapters 1-5 are preceded by the official declarations, acknowledgments, the abstract, a list of symbols used in the main text, a table of contents, and finally, a list of figures. Each chapter has its short introduction and summary, and the material is introduced in an orderly fashion.

I have a few critical comments and all of them refer to chapters 2-4.

1. p. 75, second paragraph: Two question marks instead of the number of equations.

2. p. 96, It would greatly help the reader if the differential equation for the second derivative of t with respect to the proper time be inserted somewhere on this page. Instead, we have the equation for $\ddot{\rho}$ in three different incarnations.

3. Although the caption of the Fig. 4.2 is correct, the discussion on p. 99 contains a small error: Instead of 'Two timelike geodesics, represented in the left in right figures' there should be 'Two timelike geodesics, represented in the left and central figures'.

4. The author emphasizes that the calculations have been carried out independently by him and his supervisor, using different programming languages and computational strategies, and finally compared. I presume that the results were at least similar. It would be interesting to know the criteria adopted by the author in this regard.

5. The figures illustrating the main results in the thesis are plotted for specific values of the parameters and initial conditions. My question is: What makes this choice special or typical? While one instance of an interesting numerical result may indicate the existence of an underlying phenomenon, what about its generality?

In my opinion, the results presented by Mr. Naqvi in his thesis are solid, interesting, and important, extending our understanding of the behavior of geodesics in the spacetimes of cylindrical waves. The discussion contained in Sec. 3, although inconclusive, contains some interesting ideas, which can be explored. It is worth noting that the author demonstrates

expertise in several programming languages, including C++, Python, and Mathematica. Additionally, the complexity of the problems necessitated the use of the PLgrid supercomputer for job execution.

In conclusion, the Ph.D. thesis of Mr. Syed Umair Husain contains valuable and original results and, in my opinion, satisfies all customary criteria as well as the formal requirements laid down by law. I recommend that Mr. Syed Umair Husain Naqvi should be granted permission to defend his thesis.

Podsumowując, uważam że przedstawiona mi do recenzji praca doktorska dotyczy bardzo interesującej i aktualnej tematyki. Możliwe uogólnienia i nowe kierunki badań bezpośrednio związane z przedstawionymi przez autora rezultatami również przedstawiają się obiecująco. Samą zaś pracę oceniam dobrze.

Uważam, że recenzowana praca spełnia zarówno ustawowe jak i zwyczajowe wymogi stawiane rozprawom doktorskim i wnoszę o jej przyjęcie i dopuszczenie pana mgr. Sayeda Umaira Husaina Naqviego do dalszych etapów przewodu doktorskiego.

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