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Report on the thesis *Phase structure of 1+1 Causal Dynamical Triangulation with matter* by Hongguang Zhang

The research of Hongguang Zhang is in the field of quantum gravity. The subject of his thesis is the formulation and application of a dynamical triangulation model of gravity coupled to mater. The thesis has been prepared at the Faculty of Physics, Astronomy and Applied Computer Science of the Jagiellonian University under the supervision of Jerzy Jurkiewicz.

The thesis of Hongguang Zhang is devoted to studies of quantum gravity models based on dynamical triangulations. The formulation of the theory of quantum gravity is one of the most important questions in theoretical physics. One possibility to deal with quantum fluctuations of the geometry on Planck scales is to introduce a lattice regularization. Geometries are represented by an ensemble of dynamical triangulations of the space-time. A practical possibility to impose the Wick rotation and to perform the summation over different triangulations is offered by the causal dynamical triangulation approach. The main part of the work of Hongguang Zhang is dedicated to models of matter coupled to gravity, and in particular numerical studies of dynamical triangulations. The thesis is based on four original research papers

1. A note on the Lee-Yang singularity coupled to 2-dimensional quantum gravity
   J. Ambjorn, A. Goerlich, A. Ipsen, and H.-G. Zhang, Phys. Lett. B735 (2014) 191.
2. Pseudo-topological transitions in 2D gravity models coupled to massless scalar fields
   J. Ambjorn, A.T. Goerlich, J. Jurkiewicz, and H.-G. Zhang, Nucl. Phys. B863 (2012) 421.
3. The spectral dimension in 2D CDT gravity coupled to scalar fields
   J. Ambjorn, A. Goerlich, J. Jurkiewicz, and H.-G. Zhang, e-Print: arXiv:1412.3434 [gr-qc]

4. A c=1 phase transition in two-dimensional CDT/Horava-Lifshitz gravity?
J. Ambjorn, A. Goerlich, J. Jurkiewicz, and H.-G. Zhang, Phys. Lett. B743 (2015) 435.

All the papers are published in collaboration with leading scientists in the field, from the Jagiellonian University and the Niels Bohr Institute. The thesis is supplemented with with a short overview of the field and of the obtained results. This part of the thesis is a concise introduction to lattice gravity models and the dynamical causal triangulation. The text is well written, with only few minor typos. The relations between models studied in the thesis are summarized in three diagrams.

The first publication compares the critical exponents in a continuum Liouville model and in a dimer model on euclidian dynamical triangulations. The authors demonstrate that due to operator mixing the two results agree for the magnetization exponent.

The remaining three papers of the thesis contain results of numerical investigations of scalar fields coupled to gravity. The partition function for gravity is regularized using dynamical triangulations with a finite lattice scale, and the continuum limit is not explicitly discussed. The approach is based on the causal dynamical triangulations, imposing a global foliation in the time direction. The configurations are constrained, not allowing for the creation of baby universes, which would generate singularities of the light cone structure at the point of splitting. The lattice implementation of the 2-dimensional model uses triangles with one edge or one vertex on fixed time slices. Both the time and the space boundary conditions are periodic, which gives a global torus topology. The simulations are performed with an algorithm adding or removing links on the dual lattice. In the simulations with matter, scalar field configurations are explored using a heat bath algorithm. The total number of triangles is restricted to fluctuate around a fixed value $\bar{N}$.

The pure gravity version of the model shows a broad distribution of the space volumes at different times. The dominant geometry configurations are very different in the simulation with $d > 1$ scalar fields coupled to gravity. After removing the zero mode and averaging, one observes a characteristic distribution of the space volume as a function of time. In a very restricted time domain the space volume is large (the blob). At the remaining time slices the space extend is negligible, determined by the cut-off (the stalk). The Hausdorff dimension of the blob is 3. The simulations indicate the existence of a transition between a pure gravity model and a model with matter fields.

The spectral dimension in the model with scalar fields coupled to gravity is the subject of another paper in the thesis. A diffusion process is simulated on the lattice, and the return probability is estimated. At small times, lattice artifacts are visible, and estimated by comparing diffusion with even and odd number of steps. The spectral dimension found in simulations is 2, both in the the pure gravity and the gravity+matter models. The two models show differences in the dependence on the diffusion time, reflecting different large scale topologies of the dominant configurations. *It would be interesting to discuss* *whether this value of the spectral dimension would survive if the restriction on removing baby universe creation and/or causality is removed on small scales.*

The last paper of the series is a study of a model with 4 massive scalar fields coupled to gravity. For small masses the model is expected to show similar behaviour as the in the massless case, but for large masses the model should be equivalent to the pure gravity case. By performing simulations with different masses the two regimes are clearly identified in volume profiles at different times, corresponding to a change from $d_H = 3$ to $d_H = 2$. A phase transition as a function of mass is found using a volume correlation observable with a time separation of half of the number of the maximal time extend L. In the small mass regime the correlation is small and scales as N, while in the massive regime it is large and it scales as N^2/L. The size scaling shows that the phase transition is second order or higher.

The author of the thesis has participated in a noticeable research project of 2-dimensional quantum gravity models in the causal dynamical triangulation approach, involving advanced numerical simulation techniques. The physics problems studied are relevant for the understanding of the different critical behaviours in the models. The results are the subject of 4 original research papers.

The research topic of the thesis is very actual, the work is technically sound, the author has obtained a number of specific results for models of quantum gravity. In particular, the thesis presents new results of numerical simulations of scalar fields introduced in the causal dynamical triangulation of two dimensional gravity. In summary, I find the thesis of Hongguang Zhang adequate for the PhD degree and I recommend to proceed with the oral defense of the thesis "Phase structure of 1+1 Causal Dynamical Triangulation with matter".

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